Base Systems

Introduction

The decimal system is the most widely used numerical system in society and is commonly referred to as 'base 10', given that the basic units increase by powers of ten. It is believed that this system of counting developed due to the presence of 10 digits on our hands. However, various ancient cultures used different numerical systems which still have a wide range of applications in today's world. For example, the Babylonian number system, which used 60 as its base, is still used for measuring angles, time and even geographic coordinates. Similarly, the Mayan system used base 20 and we often come across references to it in different languages – for example, "Four score and seven years ago..." in English or "quatre-vingts" ("four twenties") in French. Base systems are also important in the information technology industry. For instance, binary (base 2) serves as the basis of computer software given that it only depends on two numbers or states: 0 and 1. However, many programmers have also made use of the octal (base 8) and hexadecimal (base 16) numeric base systems to enhance data memory and storage.

Aim of workshop

The aim of this workshop is to introduce students to the fundamentals of base systems, with emphasis on binary and hexadecimal, whilst also providing insight into the specific applications of these numerical systems. This workshop should also enable students to gain a deeper understanding of how the decimal system developed, whilst also exploring different ways of counting.

Learning Outcomes

By the end of this workshop students will be able to:

- Explain, in their own words, what is meant by a base system and why we developed to using the base 10 or decimal system in everyday life
- Recognise the features of binary and hexadecimal (number of symbols, applications etc.)
- Convert a decimal number into binary or hexadecimal and vice versa
- Apply similar methods to convert a decimal number into any other base system (e.g. base 3, base 8 etc.)

Materials and Resources

(Both optional) Laminated base systems templates, whiteboard markers.

Key Words

Base system
A way of expressing numbers, using digits or other symbols, in a consistent manner.

Binary
A system of counting which has '2' as its base rather than '10'. It is also commonly referred to as 'base 2'.

Hexadecimal
A system of numerical notation that has '16' as its base (once we reach the number 16, we start back at 1 again). It is also known as 'base 16' and uses the digits 0 to 9 and the letters 'a' to 'f'.

Base Systems: Workshop Outline

SUGGESTED TIME (TOTAL MINS)	ACTIVITY	DESCRIPTION
5 mins (00:05)	**Introduction to workshop and revision of place values**	– Introduce students to the Babylonian and Mayan base systems (see **Introduction**) – Revise decimal place values with students (units, tens, hundreds, tenths, hundredths, etc.) – You may like to ask students why they think the decimal system is most common (e.g. 10 fingers)
5 mins (00:10)	**Activity 1– Place Values**	– Revise 10^0 (10 to the power of zero) – **Activity Sheet 1:** Divide students into pairs and ask them to complete Activity Sheet 1 as revision
5 mins (00:15)	**Base Systems Template**	– Introduce students to the base systems template (included below) – Demonstrate how to use the template using an example (see **Appendix – Note 1**)
10 mins (00:35)	**Introduction to Binary**	– Introduce students to binary (base 2) and its application in computer technology – Mention the number symbols in binary (e.g. 0 & 1) – Ask students "What do the symbols on a binary light switch mean?" (0 = off and 1 = on) – Show students how to convert a number from decimal into binary and vice versa using the template (see **Appendix – Note 2**)
10 mins (00:45)	**Activity 3 – Binary**	– **Activity Sheet 3**: In pairs, students attempt Activity Sheet 3 and should be encouraged to explain their thinking to one another – (Students may use a laminated template and whiteboard marker to help them. Alternatively, the template can be printed and filled in using a pencil)

SUGGESTED TIME (TOTAL MINS)	ACTIVITY	DESCRIPTION
10 mins (00:50)	**Introduction to Hexadecimal**	– Introduce students to hexadecimal (base 16) – Ask students "How many symbols do you think are required for hexadecimal?" and discuss suggestions for additional symbols that could be used (see **Appendix – Note 3**) – Discuss examples using the template – What would 16 be in hexadecimal? (This can be constructed using the template to find the answer of 10)
5 mins (01:00)	**Activity 4 – Hexadecimal**	– **Activity sheet 4:** In pairs, students attempt activity sheet 4 – Discussion of solutions with whole class

Base Systems – Workshop Appendix

Note 1: How to use the Template

Example 1: When working with place values it is often helpful to use a template. In the following example, we are using base 10:

What **number** is given here when the following three numbers are added?

- 7 x 102
- 3 x 100
- 4 x 103

1. Write the 'base number' into each of the boxes on the first row – in this case we are using base 10. Note: the indices are already included in the template.
2. Fill the given values into the second row, ensuring that they are in the correct column.
3. For each column, multiply the first and second row together and record your answer in the third row (see figure 1 below)
4. Add across all the values in the third row to get the answer i.e. 4000 + 700 + 0 + 3 = 4703

Figure 1: Diagram showing how to use base systems template for decimal values

10^5	10^4	10^3	10^2	10^1	10^0	
		4	7	0	3	
		4×10^3 =4000	7×10^2 =700	0×10^1 =0	3×10^0 =3	Total 4703

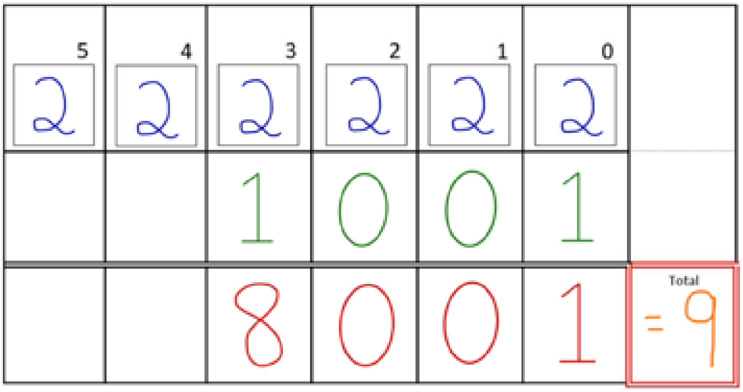
Note 2: How to Convert a Number from Binary to Decimal and Vice Versa

Example 1: When converting a number from binary to decimal it is often helpful to use a template. In the following example, we are converting the binary number "1001" into decimal.

1. Fill in the 'base number' into each of the boxes on the top row (in this case we are using base 2)
2. Write the binary number that you wish to convert to decimal into the second row, making sure to separate the digits (e.g. 1001)
3. Multiply the top two rows together to get the third row e.g. 20 x 1 = 1, 21 x 0 = 0 etc.
4. Add across each of the boxes in the final row to get the converted decimal answer e.g. 8 + 0 + 0 + 1 = 9

5 The resulting decimal answer is therefore 9

Figure 2: Diagram showing how to convert from binary to decimal using the template



Example 2: When converting from decimal to binary, we need to work backwards and think about what 'base 2' values make up the decimal number when added together. In the following example, we are converting the decimal number '6' into binary.

1. Fill in the 'base number' into each of the boxes on the top row (in this case we are going to convert to base 2)
2. Ask yourself "what base 2 values make up the given decimal number?"

   E.g. In the case of the decimal number 6, we know that anything above 23 (=8) is too large. We therefore look at the remaining base 2 values in the template:

   22 = (4) 21 = (2) 20 = (1)

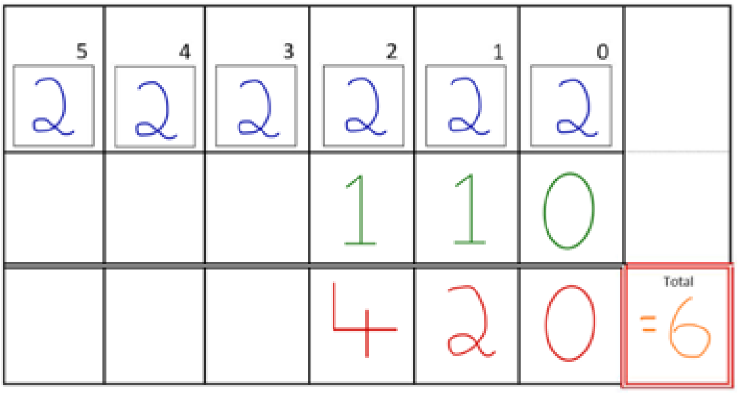
   By only taking one or none of the above values, how can we make 6? Trial and error is the best way forward.

   Well, we could take one 22, one 21 and zero 20 which would equal 6 when added together i.e. 4 + 2 + 0 = 6. We can fill this into our template (as shown below).

   Note: it is important to include the 0 under 20 given that the binary number '110' is very different to the binary number '11'.
3. The answer can be checked by multiplying the top row by the second row and adding to see if we get 6.

4. The decimal number 6 is thus 110 in binary.

Figure 3: Diagram showing how to convert from decimal to binary using the template

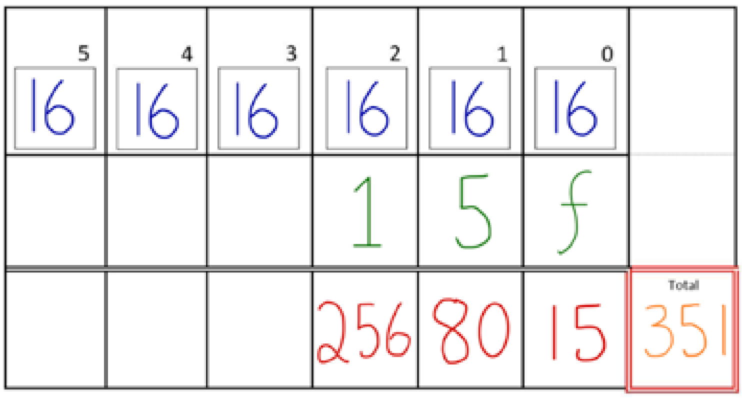


Hexadecimal uses 16 distinct symbols, however, we are only familiar with our distinct numbers 0 to 9. Therefore, additional symbols, in this case a to f, are used to represent the decimal values 10 to 15.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f

When converting a number from hexadecimal to decimal we can use the same method as shown in Note 2, Example 1. For instance, the hexadecimal value '15f' would be 351 in decimal (see figure 4).

Figure 4: Diagram showing how to convert from hexadecimal to decimal using the template



Sources and Additional Resources

http://www.binaryhexconverter.com/decimal–to–binary–converter (Decimal to Binary Converter)

http://www.binaryhexconverter.com/decimal–to–hex–converter (Decimal to Hexadecimal Converter)

http://www.purplemath.com/modules/numbbase.htm

http://www.storyofmathematics.com/mathematicians.html

Base Systems Activity 1

Place Values

1. **What number is given here when added?**
   - 7 hundreds
   - 3 units
   - 4 thousands

2. **What number is given here when added?**
   - 5 units
   - 1 ten thousand
   - 3 hundreds

3. **What number is given here when added?**
   - 9 tens
   - 2 thousands

4. **What number is given here when added?**
   - 5 x 101
   - 9 x 104
   - 1 x 102

5. **What number is given here when added?**
   - 6 x 100
   - 7 x 103
   - 2 x 101

6. What number is given here when added?

- 4 x 105
- 8 x 100
- 7 x 101

7. Describe, in your own words, how changing the powers of 10 changes the values of the number.

8. Do we use base 10 for all of our everyday numerical dealings? Describe examples where we use other bases.

Base Systems Activity 2

Binary (Base 2)

1. **The following numbers are written in binary form (base 2). Can you convert them into decimal (base 10)? You may use the template provided to help you.**

(a) 111

(b) 11011

(c) 10110

Base Systems Activity 2

2. **If we work the other way, can you translate these decimal numbers into binary? You may use the template provided to help you.**

(a) 9

(a) 24

(a) 37

Base Systems: Activity Sheet 2

3. In base 10, **Lucy** is 16 years old. In base 2, **Patrick** is 1101 years old? Who is the eldest?

4. Come up with some challenging number conversions for the person beside you.

Base Systems Activity 3

Hexadecimal (Base 16)

1. Can you calculate your **age** in hexadecimal?

2. What would today's **date** be in hexadecimal?

Base Systems – Blank Template

5	4	3	2	1	0	
						Total